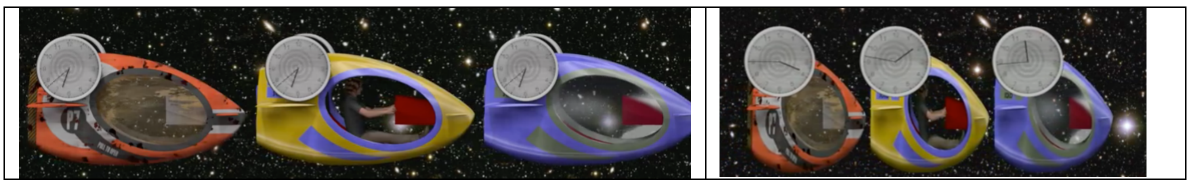
## Khutoryansky Video: The Relativity of Time and Simultaneity

Here are the Minkowski Space-Time diagrams for two of the sequences in the video "Nature of Time and Simultaneity" by Eugene Khutoryansky, at https://www.youtube.com/watch?v=ruRrVWHcgws

First, starting at 1:33, we have three spaceships moving together at the same speed, meaning they are all in the same frame of reference. Therefore an observer on any of the spaceships sees clocks all show the same time. But a stationary observer sees the (length contracted) spaceships showing different times, with the latest time on the left and the earliest time on the right.

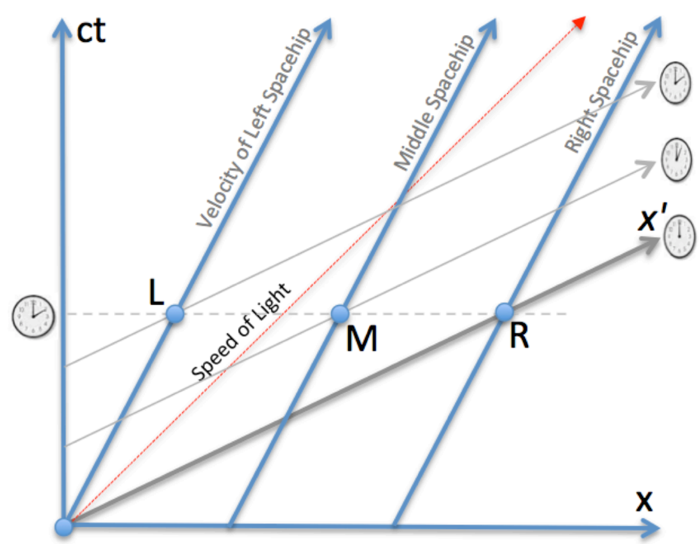


We now show how this looks from the point of view of the stationary observer on the space-time diagram. The three spaceships have the same velocity, but are in different locations, so their locations are described by three parallel world lines.

The ***x'*** axis is the space axis for all three spaceships. The time along this axis is the same for all the spaceships, which we set to 0:00, as shown by the contracted clocks on the right. The two lines parallel to this axis all have the same time (1:00 and 2:00) in the frame of reference of the spaceships.

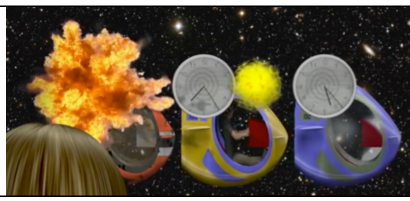
The stationary observer is seeing all three spaceships at the same time (2:00) in her frame of reference, as shown by the horizontal dashed line, parallel to her **x** axis.

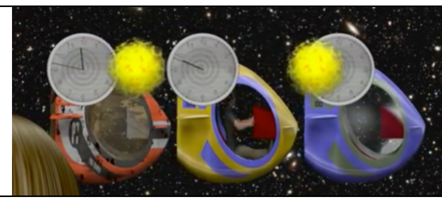
Events **L**, **M** and **R** show where the spaceships are located at this time. We can read the spaceship times using the sloped lines of simultaneity, and see that the times on the spaceships are 2:00, 1:00 and 0:00 respectively.



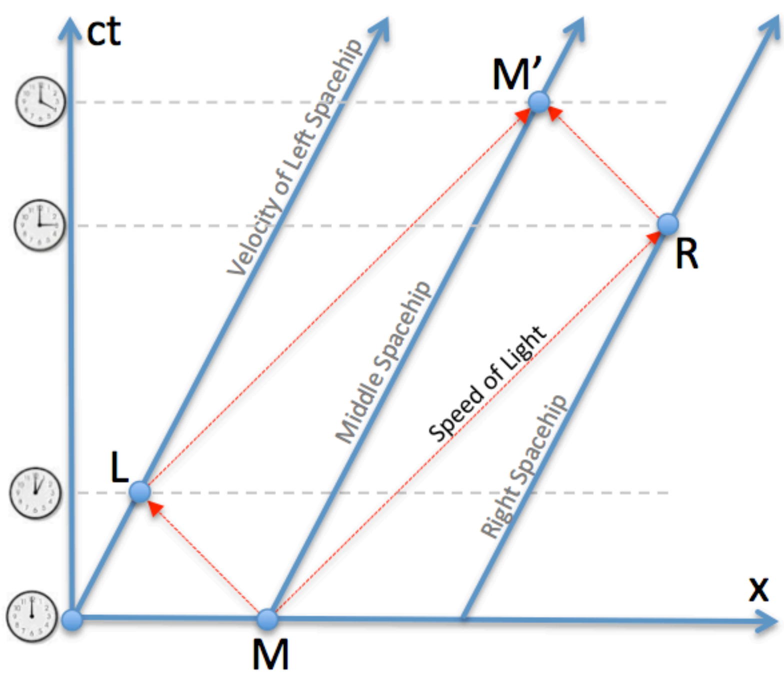
In the next sequence, starting at 4:37, the middle spaceship fires a photon at the speed of light toward the left and right spaceships, who both immediately reflect it back. In the frame of reference of the spaceships, the photons arrive at the left and right spaceships at the same time, and both reach the middle spaceship later at the same time.

From the outside observer's point of view, the photon arrives at the left spaceship before the right spaceship. However, both the ejection of both photons, and their simultaneous arrival back at the middle spaceship, are each single events, which must be seen to happen at the same time by all observers.



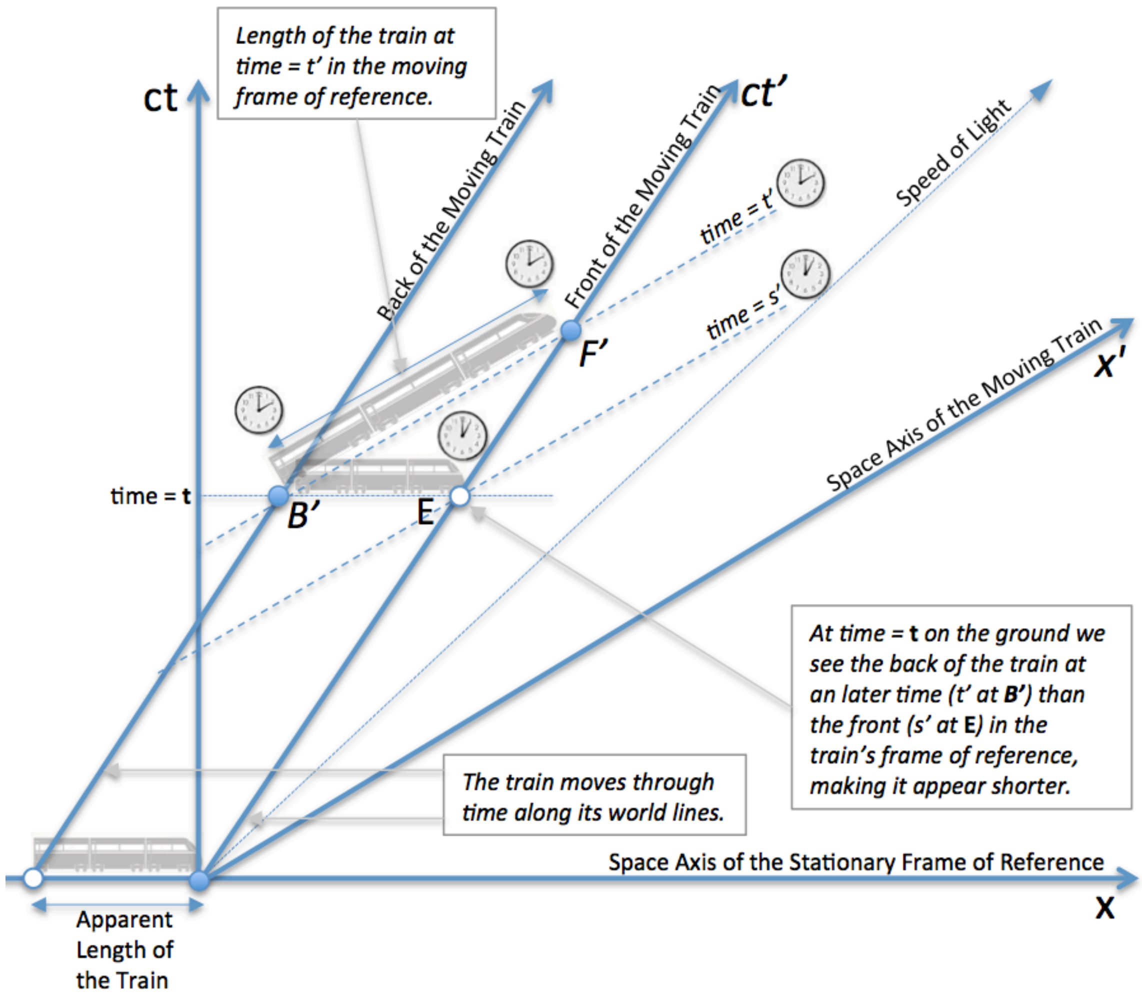


This time in the space-time diagram, we are looking at the dashed lines of time simultaneity for the stationary observer. At event **M** (0:00 in the stationary frame of reference) we observe both photons ejected from the middle spaceship. As the speed of light is the same in every frame of reference, these photons are shown as travelling at 45° on the diagram. At event **L** (1:00) we see the photon reach the left spaceship. At event **R** (3:00) we see the photon reach the right spaceship. Finally, at event **M'** (4:00) both photons arrive back at the middle spaceship.



## Apparent Length Contraction of a Relativistic Train

A train is moving at a relativistic speed relative to a stationary observer. The train cannot be treated as a single event because it exists across multiple locations in space. Therefore the motion of the front and back of the train is shown with two separate world lines. The train is moving from left to right, and the stationary observer encounters the front before the back along any vertical time axis.



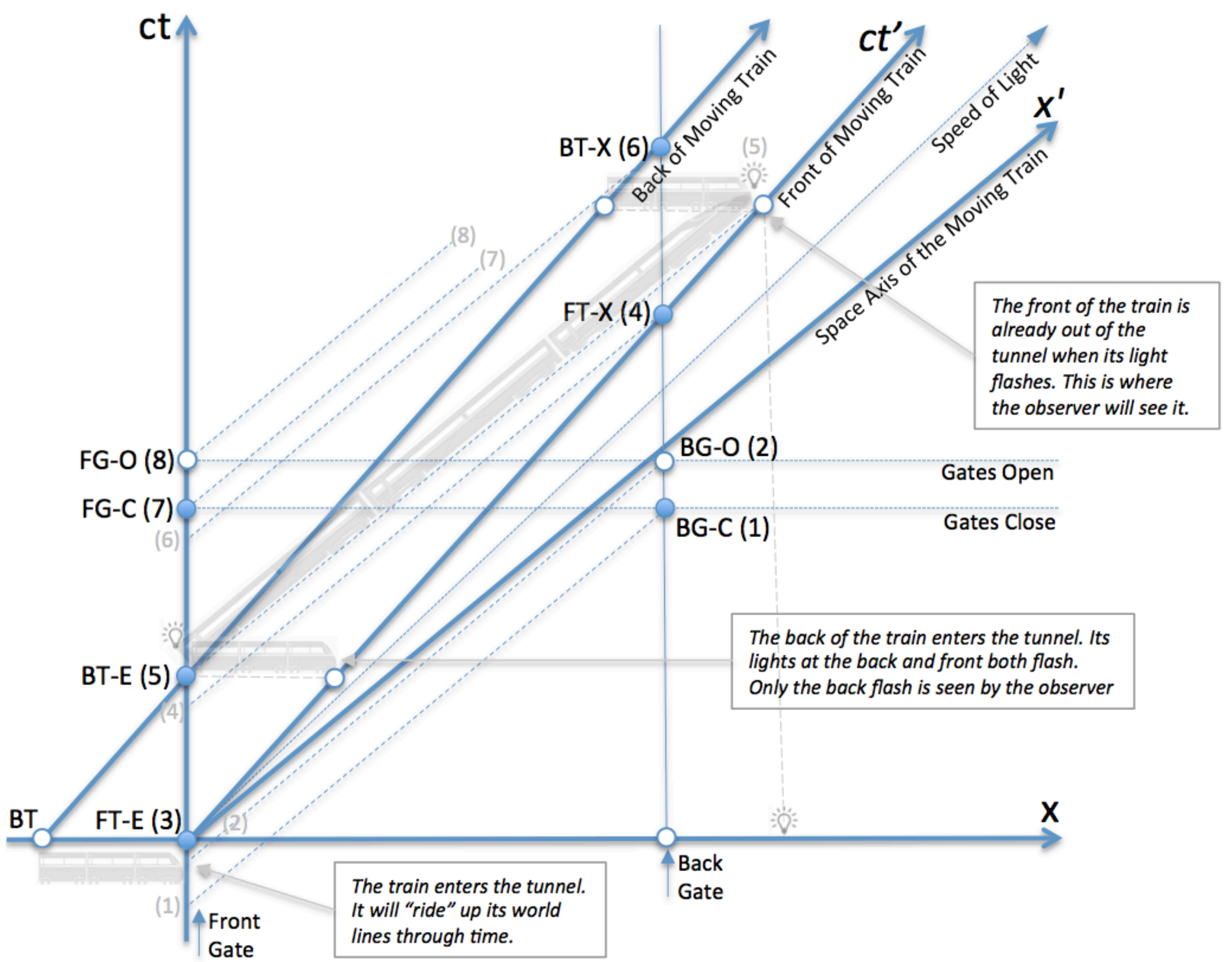

Events **B** and **F** mark the back and front of the train at time $t'$ in the moving frame of reference. The length of the train is the distance between **B** and **F**. The stationary observer observes the event **B**, the back of the train, at time "**t**", and therefore must see the front of the train at the same time "**t**", parallel to the x-axis, which is event **E**. We are seeing the front of the train at an earlier time ($s'$) than the back ($t'$) in the train's frame of reference. The front has not had time to "catch up" to where it should be compared to the back, and that is why the train seems shorter. The distance in space between events **B** and **E** is the apparent length of the train.

## Khutoryansky Video: A Relativistic Train Passing Through a Tunnel

In the Khutoryansky video (https://www.youtube.com/watch?v=Xrqj88zQZJg) "Einstein's Tunnel Paradox", a train travelling near the speed of light passes through a short tunnel with gates on both ends. The stationary observer sees the train's length as shortened, so it fits in the tunnel. Both gates close simultaneously when the train is entirely inside the tunnel, and then open again before it leaves.

In the moving train's frame of reference, the train is longer than the tunnel. Both observers must see the train enter and leave the tunnel, and observe the same gate opening and closing events. The gates must not close while the train is over them. However, the moving train experiences these events in a different order.

The train cannot be treated as a single event because it exists across multiple locations in space. Therefore the motion of the front and back of the train must be shown with two separate world lines. The time experienced by the train is defined by lines parallel to its space ($x'$) axis, shown here as dashed lines with sequence numbers.



The two vertical world lines represent the space dimension of the front and back gates of the tunnel in both frames of reference. All the events (shown by dots with three-letter labels) take place on these world lines.

In the stationary frame of reference, simultaneous events in time are horizontal lines parallel to the x-axis. The events happen in the following order:

| | |
|---|---|
| **FT-E** | The front of the train enters the tunnel. |
| **BT-E** | The back of the train enters the tunnel. |
| **FG-C**, **BG-C** | The front and back gates close, enclosing the train in the tunnel. |
| **FG-O**, **BG-O** | The front and back gates open while the train is still in the tunnel. |
| **FT-X** | The front of the train exits the tunnel through the open gate. |
| **BT-X** | The back of the train exits the tunnel. |

When the train is fully inside the tunnel at event **BT-E**, the back and front of the train as seen in the stationary frame of reference are shown where the horizontal time line at those events intersects the world lines of the back and front of the moving train. The train easily fits within the tunnel.

In the moving train's frame of reference, simultaneous events in time occur on the dashed lines parallel to the $x'$ axis. All event lines are projected to the front gate (on the stationary time axis) with their order indicated with numbers in brackets.

From the train's point of view, the events occur in the following order:

| | |
|---|---|
| **BG-C**(1) | The back gate closes before the train arrives. |
| **BG-O**(2) | The back gate opens in time for the train to arrive. |
| **FT-E**(3) | The front of the train enters the tunnel through the open gate. |
| **FT-X**(4) | The front of the train exits the tunnel before the back enters. |
| **BT-E**(5) | The back of the train now enters the tunnel. |
| **BT-X**(6) | The back of the train exits the tunnel. |
| **FG-C**(7) | The front gate finally closes, after the train has left. |
| **FG-O**(8) | The front gate opens again. |

The moving train is shown at event **BT-E**(5), as the back of the train has just entered the tunnel. The front of the train is already well past the back gate at the tunnel exit.

Imagine that the moving train blinks lights on its back and front the moment its back enters the tunnel at event **BT-E**. The stationary observer will see only the back light blink at this moment (event). The front light will be seen to blink much later, after the front of the train is out of the tunnel. The blinking front light event must happen in the same place in all frames of reference.